Reply to "Comment on 'Power-law correlations in the southern-oscillation-index fluctuations characterizing El Niño' "

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Earlier [Phys. Rev. E **63**, 047201 (2001)] we studied the southern oscillation index (SOI). Our findings tended to favor specific physical models for the El Niño description. The Comment by Metzler [Phys. Rev. E **67**, 018201 (2003)] on this publication does not give any argument in favor of another El Niño physical model. In contrast, the Comment points out that statistical properties of the SOI data can be explained with a model based on a linear autoregressive process, but such a modeling does not help in identifying the relevant physical mechanisms.

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In Ref. [1] we studied the southern oscillation index (SOI) during 1866–2000. An anomalous tail of the cumulative distribution of the fluctuations of the SOI signal was found with an occurrence of extreme events. Large fluctuations occur more often than the Gaussian distribution would predict. The signal energy spectrum was analyzed and the Detrended Fluctuation Analysis (DFA) performed on the SOI signal. Self-affine properties and power law correlations in the signal fluctuations were suggested. Antipersistent type of correlation exists for a time interval ranging from about 4 months to about 6 years. This tends to favor specific physical models for the El Niño description.

Metzler [2] points out that the statistical properties of the SOI data as analyzed in Ref. [1] could be explained with a model based on a linear autoregressive process. Even though it is not his aim; "to show that all statistical properties of the SOI data can be explained with this sort of model," yet some SOI properties can be found in short-range correlated artificial data, he writes.

Many authors search for universal laws, under the form of power laws in order to test the scaling hypothesis and hierarchical structure in systems and phenomena. At this time this procedure calls for accumulation of data and empirical analyses that do not always follow rigorous statistical procedures. Of course, we concur with Metzler that "statistical properties of a time series should (. . .) be compared to possible models to make any meaningful statement . . . *when appropriate models exist*." (*Our emphasis*.) It is recognized that a law with a single power exponent consists of a first-order approximation. Macai *et al.* [3] pointed out that a straight line can fit many types of curves on a log-log plot. Therefore we concur that one should not stretch the discussion presented in Ref. [1] to much.

One might debate which of two URL addresses provided as references for the source of the data should be used. We have used the data set listed in ASCII format with eight decimal point precision at the URL in Ref. [4] (thus 1602 data points) and data from the URL in Ref. [5] from July 1999 to April 2000 (thus taking into account extra data points).

It seems that Metzler [2] analyzes a data set with one decimal point precision presented from the URL in Ref. [6]; a disagreement is then found on the values for the μ exponents for the same range of $|x|$ values. The scaling is not perfect, and it was never claimed otherwise in Ref. [1]. The scaling should be much better from a model data series that can be as long as desired; note that Metzler did not propose such a comparison establishing the intrinsic features of his so-called model. Nevertheless the error bars we quoted are quite satisfactory for this type of work and for its purpose. The linear fit within the error bars is aimed to test our hypothesis whether the distribution is Levy stable or not. The values of μ exponent *confirm* a lack of Gaussian law in the region specified. The lack of error bars in the Comment by Metzler [2] does not allow a comparison of confidence intervals from both analyses. In Ref. [1] it was written that the statistics is not sufficient for large amplitudes of the fluctuations.

In addition to Ref. [1] we tested the fluctuation distribution with a Kolmogorov-Smirnov procedure on two types of surrogate data: one in which the amplitudes are randomly shuffled and the other where the sign of the SOI signal is randomly shuffled. It is known that [7] the origin of broad-tailed distributions is a key question; the broad tails are thought to be caused by long-range volatility correlations. This was the interesting question for our SOI analysis. Destroying all correlations by shuffling the order of the fluctuations is known to cause the broad tails almost to vanish. Destroying only sign correlations, by shuffling the order of only the signs (but not the absolute values) of the fluctuations, allows the broad tails to persist. We did not find it worthwhile to add an extra figure that would only prove the expected or even obvious.

One can surely find many mathematical functions beside the one mentioned in the preceding comment, functions which have a power spectrum with the same slope as the SOI signal of Ref. [1]. However, it is well known [8–11] that a power law scaling of the power spectrum of the process is a necessary condition for self-affinity. However, it is also well known that the Fourier transform technique is not very pre-

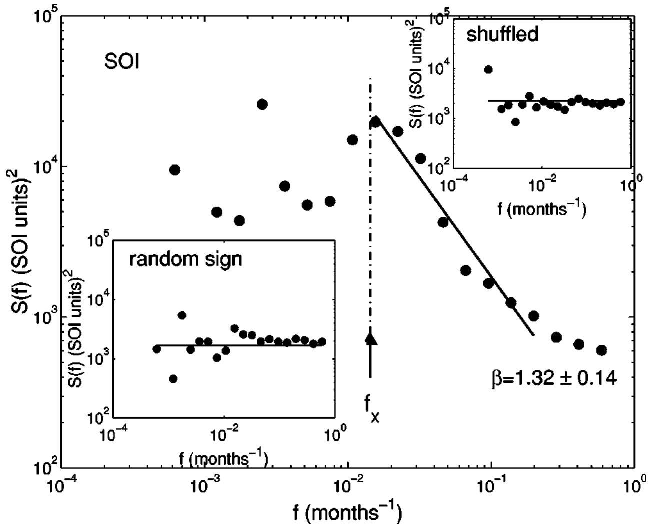

FIG. 1. The energy spectrum of the SOI data (from Fig. 1 in Ref. [1]). A spectral exponent $\beta=1.32\pm0.14$ characterizes the correlations of fluctuations in the frequency range from about 1/5 to about 1/64 $(\text{month})^{-1}$. Insets: The energy spectrum of the shuffled (upper inset) and random sign (lower inset) SOI signal.

cise in determining the scaling range and the spectral exponent. It depends on the bin size distribution, which is used for the fit. Therefore, as it is written in Ref. [1]: ''To better estimate the crossover and to test the correlations using a different approach, we analyze the SOI signal applying the DFA technique.'' Our statement about the lack of fully interesting information from the power spectrum is turned by Metzler [2] into a criticism.

In fact, in order to convince oneself about the statistical significance, some extra tests are useful. Since there is no other data from the same experiment, the surrogate data technique can be used for subsequent comparison. After several such Monte Carlo simulations, a (trivial) Brownian noise behavior was found in the surrogate data. When keeping the amplitude but stochastically reversing the sign, again after several Monte Carlo simulations, we obtained similar Brownian noise behavior. The energy spectrum of the SOI data, from Fig. 1 in Ref. [1] leads to a spectral exponent $\beta=1.32\pm0.14$ characterizing the correlations of fluctuations in the frequency range from about 1/5 to about 1/64 $(\text{month})^{-1}$. For comparison, we show in the insets of Fig. 1 the energy spectrum of the shuffled (upper inset) and random sign (lower inset) SOI signal. We conclude that the found slope of the SOI signal in Ref. [1] is significantly outside the error bars of the surrogate data corresponding values. By the way, each comparison used the Kolmogorov-Smirnov testing procedure.

The DFA function of the test data in Fig. 5 of the comment shows three, not two, possible scaling regions as Mezler [2] states. The DFA function of the SOI signal in this Fig. 5 is very different from the DFA function that we obtain following the same steps of the analysis. We relate this to the fact that Metzler [2] uses a different data set.

An interesting point concerns the ''missing first step.'' For this Reply, we have performed the analysis on the SOI signal time series that is integrated (so-called first step) to mimick a random walk

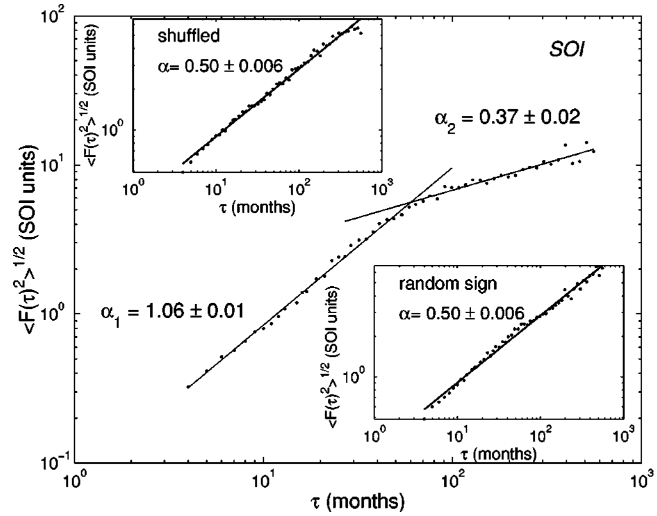

FIG. 2. The DFA function $\langle F^2(\tau)\rangle^{1/2}$ in log-log plot for the SOI data from Fig. 1 in Ref. [1]. Two scaling regimes are observed, $\alpha_1=1.06\pm0.01$ and noiselike $\alpha_2=0.37\pm0.02$ with a crossover at ≈ 64 months.

$$Y(n)=\sum_{i=1}^{n}\,[y(i)-\langle y\rangle],$$

where $\langle y\rangle=\Sigma_{i=1}^{N}y(i)/N$.

The DFA function as shown in Fig. 2 clearly possesses two scaling regions characterized by $\alpha_1=1.06\pm0.01$ and $\alpha_2=0.37\pm0.02$ with a crossover at about 64 months. In the insets the DFA functions of the two types of surrogate data we used are also shown. Both of them scale as short-range correlated data with an exponent 1/2 as expected and confirm the analysis in Ref. [1], without adding anything to the fact that the SOI signal fluctuations can be both medium- and long-range correlated. We feel that a periodic pattern leads to a crossover in the DFA graph, as shown by Hu *et al.* [12]

Thus, it can be considered that the findings of the long-range correlations in the SOI signal in Ref. [1] are robust. This confirms the relevance of the measurements of a signal precision.

Regarding the many statements by Metzler on analyzing data with care, we wholly agree, but have never done otherwise. We never wrote that scaling laws hold everywhere; we agree that an artificial model is sometimes statistically better than a physical one for explaining a phenomenon.

However, there are many models that aim to forecast the El Niño phenomena based on the canonical correlation analysis or a geophysical model, for example, Refs. [13–17].Other types of forecasting models are various empirically derived or inverse models as the one suggested in Ref. [18] and the method based on the Fokker-Planck equation for the probability distribution function derived directly from the observation data and the Langevin equation for the evolution of the signal [19]. We are aware also of models that consider the El Niño phenomena as a stochastically driven process and as a composite of a few oscillating modes [20–22].

There is a misprint in Ref. [1] p. 047201-2: ''$\mu=3.30\pm0.06$ when the amplitudes of fluctuations are between 1.5 and 2.8'' should be read: ''$\mu=3.30\pm0.06$ when the amplitudes of fluctuations are between 1.2 and 2.8.''

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